Adaptive Prediction for Weakly Dependent Functional Time Series

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Functional Time Series (FTS) commonly arise in applications where data are collected over time as curves,

- Environmental science, Clinical research, Sports, Finance, etc.
- Energy

FTS consists of N serially-dependent curves $X_1, \dots, X_n, \dots, X_N$, defined over a continuous domain $I = (0, 1]$.

- The index n may represent the day, while I corresponds to the rescaled daily clock time.

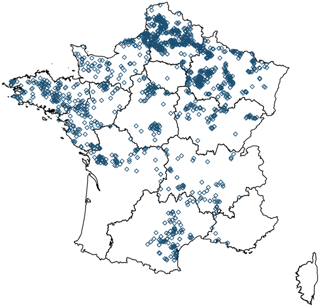
This work has been part of Datastorm's R&D activity on energy data.

Practical purposes :

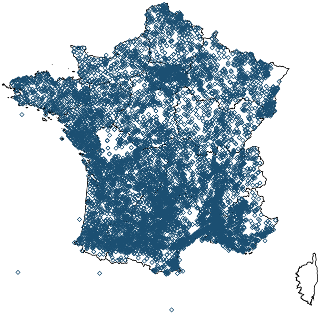
- Accurate forecasts guide energy purchasing and revenue planning.
- The growing share of renewable energy increases variability, making accurate forecasts essential.

Data : over 1,200 wind and 7,000 PV farms connected to the Enedis network in 2021, with production described by daily load-factor curves.

(a) Location of wind farms



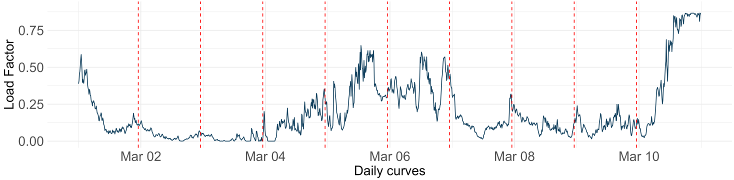
(b) Location of photovoltaic farms



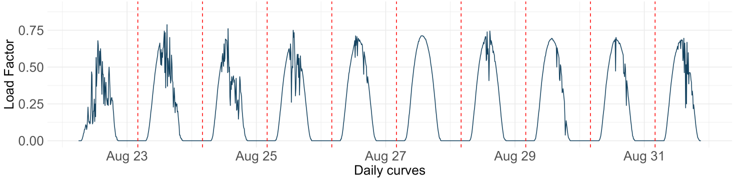
Wind and PV electricity production

- A set of N serially-dependent daily curves X_n.

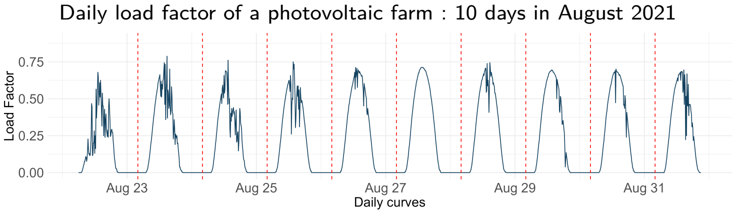
(a) Daily load factor of a wind farm : 10 days in March 2021



(b) Daily load factor of a photovoltaic farm : 10 days in August 2021



Irregular curves and discrete observations



- The trajectories are **irregular**.
- The curves are observed at **discrete sets of points**.
- **Measurement errors**.
- Two different observation schemes may occur :
  - Common design.
  - Random or independent design (the focus today).

The data

Data points $(Y_{n,k}, T_{n,k})$, with $T_{n,k}$ domain points.

The model :

$$Y_{n,k} = X_n(T_{n,k}) + \sigma(T_{n,k})\varepsilon_{n,k}, \qquad 1 \leq k \leq M_n, \quad 1 \leq n \leq N.$$

Assumptions :

- $\{X_n\}$ is a stationary process of $\mathcal{H} = \mathbb{L}^2(I)$,
- $M_1, \ldots, M_N \overset{i.i.d.}{\sim} M$, taking integer values $M \geq 2$ with $\mathbb{E}[M] = \lambda$,
- $T_{n,k} \overset{i.i.d.}{\sim} T$, with a strictly positive density,
- $\varepsilon_{n,k} \overset{i.i.d.}{\sim} \epsilon$, a centred unit-variance error ; $\sigma^2(\cdot)$ is Lipschitz continuous.
- $\{X_n\}$, $\{M_n\}$, $\{\varepsilon_{n,k}\}$, and $\{T_{n,k}\}$ are mutually independent.

Motivation

We aim to build a procedure for **curve prediction** that adapts to the **local regularity** of the trajectories for FTS in the context of **weak dependence**.

Using **discrete and noisy data**, our goal is to perform **adaptive estimation** of :

- the Best Linear Unbiased Predictors (BLUP) that is a combination of
- mean, covariance and autocovariances functions.

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- the Best Linear Unbiased Predictors (BLUP) that is a combination of
- mean, covariance and autocovariance functions.

The proposed procedure consists of the following steps :

Step 1 : estimate the local regularity parameters of the trajectories

Step 2 : estimate the mean & (auto)covariances functions

Step 3 : build BLUP predictor using these estimates.

Step 4 : application to Wind and PV electricity production forecasting

Outline

Local regularity parameters

Let X denote a generic random function with the stationary law of $\{X_n\}$, and **continuous** and **non-differentiable** sample paths.

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Definition (Golovkine *et al.*, 2022)

The random function X admits a ***local regularity*** at $t \in I$, with local exponent $H_t \in (0,1)$ and local Hölder constant $L_t > 0$, if $\Delta > 0$ exists such that

$$\mathbb{E}\left[\{X(u) - X(v)\}^2\right] \approx L_t^2 |u - v|^{2H_t},$$

with $t - \Delta/2 \leq u \leq t \leq v \leq t + \Delta/2$.

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- H_t and L_t^2 are the ***local regularity parameters***.

Examples

1. Let B^H be a **fractional Brownian motion**, with Hurst index $H \in (0, 1)$,

$$\mathbb{E}\left[\left\{B^H(u) - B^H(v)\right\}^2\right] = |u - v|^{2H}, \quad u, v \in \mathbb{R}_+.$$

Here, the local regularity parameters are constant : $H_t \equiv H$ and $L_t^2 \equiv 1$.

2. **Multi-fractional Brownian motion** (MfBm) with a twice continuously differentiable **Hurst function**.

3. **Functional autoregressive process of order 1** (FAR(1))

$$X_n(t) = \mu(t) + \int_0^1 \psi(t, s)\{X_{n-1}(s) - \mu(s)\}\, ds + L_t \xi_n(t),$$

with mean μ and driven by i.i.d. MfBm white noise $\{\xi_n\}$.

Nonparametric estimation of local regularity parameters

For u, v close to $t \in I$, let

$$\theta(u,v) = \mathbb{E}\left[\{X(u) - X(v)\}^2\right] \approx L_t^2 |u - v|^{2H_t},$$

with empirical counterpart

$$\widehat{\theta}(u,v) = \frac{1}{N}\sum_{n=1}^{N}\{\widetilde{X}_n(v) - \widetilde{X}_n(u)\}^2,$$

where $\widetilde{X}_n$ is a nonparametric estimator of X_n obtained from $\{Y_{n,k}, T_{n,k}\}_k$.

For $t_1 = t - \Delta/2$ and $t_3 = t + \Delta/2$, the local regularity estimators are :

$$\widehat{H}_t = \frac{\log(\widehat{\theta}(t_1,t_3)) - \log(\widehat{\theta}(t_1,t))}{2\log 2},$$

$$\widehat{L}_t^2 = \frac{\widehat{\theta}(t_1,t_3)}{\Delta^{2\widehat{H}_t}}.$$

Weak dependency assumption

For a local study, the dependency assumption on $\{X_n\}$ should be inherited by $\{X_n(t)\}$, for all $t \in I$.

We use an extention [1] of the Hörmann & Kokoszka (2010) weak dependence to $(\mathcal{C}, \|\cdot\|_\infty)$-valued processes $\Rightarrow$ new concept : *$\mathbb{L}^p_{\mathcal{C}}$-m-approximability*.

From now on, we assume $\{X_n\}$ is *$\mathbb{L}^p_{\mathcal{C}}$-m-approximable*, with $p \geq 4$.

Example : The FAR(1) from the previous example is *$\mathbb{L}^p_{\mathcal{C}}$-m-approximable*.

Asymptotic properties [1] *:* concentration bounds for $\widehat{H}_t$ and $\widehat{L}_t^2$.

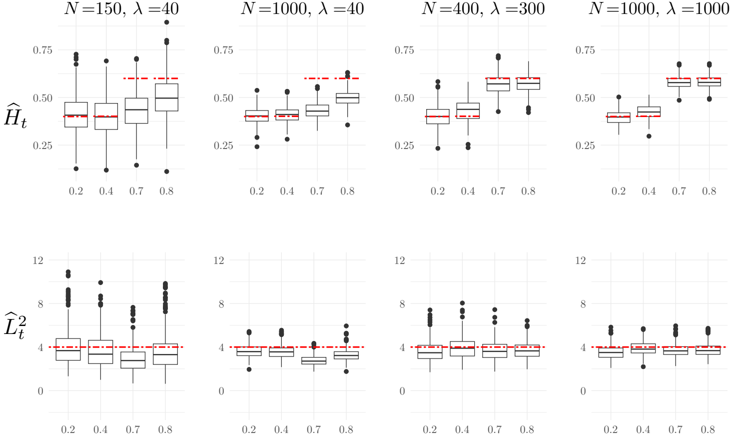
1. see Maissoro et al. (2025)

Numerical illustration : estimation of H_t and L_t^2

Settings : FAR(1) with MfBm WN with logistic shape Hurst function, $L_t^2 = 4$:

- $\mu(t) = 4\sin(3\pi t/2)$, $\psi(t,s) = \kappa \exp\left(-(t+2s)^2\right)$, noise std $\sigma = 0.25$.

Estimates at $t \in \{0.2, 0.4, 0.7, 0.8\}$ from 400 replications.



Outline

Step 2 : Adaptive mean and autocovariance estimators for FTS

Let $s, t \in I$ and lag $\ell \geq 0$:

$$\mu(t) = \mathbb{E}[X_n(t)] \qquad \text{mean function,}$$
$$\Gamma_\ell(s,t) = \mathbb{E}[\{X_n(s) - \mu(s)\}\{X_{n+\ell}(t) - \mu(t)\}] \qquad \text{autocovariance function.}$$

Goal : locally adaptive estimation of the **mean** and **(auto)covariances** of stationary, **weakly dependent** FTS, using **kernel smoothing**.

Related work :

- Rubin & Panaretos (2020) – FTS : pool all data points and assume μ and Γ_ℓ are at least twice differentiable.
- Golovkine *et al.* (2025) – i.i.d. functional data : '*smooth first, then estimate*' estimators of μ and Γ_0 that adapt to the local regularity.

Adaptive estimation of μ

The proposed estimator is $\widehat{\mu}_N^\star(t) = \widehat{\mu}_N(t; h_\mu^\star)$, with

$$\widehat{\mu}_N(t; h) = \sum_{n=1}^{N} \frac{\pi_n(t; h)}{P_N(t; h)} \widehat{X}_n(t; h), \qquad \text{where}$$

- $\pi_n(t; h) = 1$ if at least one $T_{n,i} \in [t-h, t+h]$, 0 otherwise,
- $P_N(t; h) = \pi_1(t; h) + \ldots + \pi_N(t; h)$, $P_N(t; h) \leq N$
- $\widehat{X}_n(t; h)$ is Nadaraya-Watson estimator,

where $h_\mu^\star := h_\mu^\star(t)$, is an adaptive, optimal bandwidth,

$$h_\mu^\star \in \underset{h \in \mathcal{H}_N}{\arg\min}\ \widehat{R}_\mu(t; h, \widehat{H}_t, \widehat{L}_t^2, \widehat{\sigma}^2(t)), \quad \text{with } \mathcal{H}_N \text{ a bandwidth set, and}$$

$$\mathbb{E}_{M,T}\left[\{\widehat{\mu}_N(t; h) - \mu(t)\}^2\right] \leq R_\mu(t; h, H_t, L_t^2, \sigma^2(t)), \quad \text{where}$$

$$R_\mu(t; h, H_t, L_t^2, \sigma(t)) = \text{Squared Bias} + \text{Variance} + \text{Penalty}.$$

Adaptive estimation of Γ_ℓ

The adaptive estimator of $\Gamma_\ell(s,t)$ is $\widehat{\Gamma}^\star_{N,\ell}(s,t) = \widehat{\Gamma}_{N,\ell}(s,t; h_s^\star, h_t^\star)$, with

$$\widehat{\Gamma}_{N,\ell}(s,t;h_s,h_t) = \sum_{n=1}^{N-\ell} \frac{\pi_n(s;h_s)\pi_{n+\ell}(t;h_t)}{P_{N,\ell}(s,t;h_s,h_t)} \widehat{X}^c_n(s;h_s)\widehat{X}^c_{n+\ell}(t;h_t),$$

- where $\widehat{X}^c_n(u;h) = \widehat{X}_n(u;h) - \widehat{\mu}_N(u;h)$ and,
- $P_{N,\ell}(s,t;h_s,h_t)$ is the sum of $\pi_n(s;h_s)\pi_{n+\ell}(t;h_s)$, $P_{N,\ell}(s,t;h_s,h_t) \leq N$.

Here, $(h_s^\star, h_t^\star) = (h_\ell^\star(s|t), h_\ell^\star(t|s))$ are adaptive, optimal bandwidth, chosen to minimise a bound on the quadratic risk :

$$(h_s^\star, h_t^\star) \in \underset{(h_s,h_t)\in\mathcal{H}_N\times\mathcal{H}_N}{\arg\min} \widehat{R}_{\Gamma_\ell}(s,t;h_s,h_t),$$

$$\text{with} \quad R_{\Gamma_\ell}(s,t;h_s,h_t) = \text{Squared Bias} + \text{Variance} + \text{Penalty},$$

$$R_{\Gamma_\ell}(s,t;h_s,h_t) := R_{\Gamma_\ell}(s,t;h_s,h_t,H_s,H_t,L_t^2,L_s^2,\sigma^2(s),\sigma^2(t)).$$

Asymptotic properties

Pointwise consistency results :

$$\widehat{\mu}_N^{\star}(t) - \mu(t) = \mathcal{O}_{\mathbb{P}}\left((N\lambda)^{-H_t/(1+2H_t)} + N^{-1/2}\right).$$

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Uniform consistency results : for any $\underline{H} \in [2/(p-4), \inf H_t)$,

$$\|\widehat{\mu}_N^{*} - \mu\|_{\infty} = \mathcal{O}_{\mathbb{P}}\left(\left\{\frac{\log(N\lambda)}{N\lambda^{2-2/p}}\right\}^{\underline{H}/\{2\underline{H}+2-2/p\}} + \sqrt{\frac{\log(N\lambda)}{N}}\right).$$

- $p > 4 + 2/\inf H_t$, denotes the moment order in $\mathbb{L}^p_{\mathcal{C}}$-$m$-approximability.
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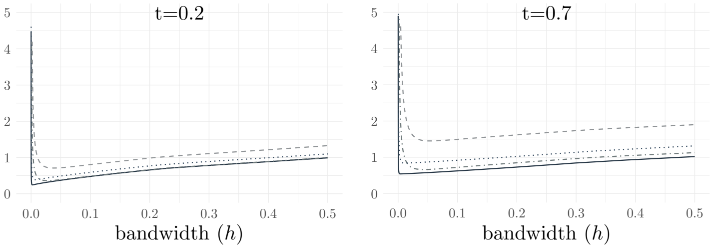
Similar consistency results have been established for the lag-ℓ autocovariance.

Numerical illustration : mean function estimation

Settings : FAR(1) with MfBm WN with logistic shape Hurst function, $L_t^2 = 4$:

- $\mu(t) = 4\sin(3\pi t/2)$, $\psi(u,s) = \kappa \exp\left(-(u+2s)^2\right)$, noise std $\sigma = 0.25$.

Empirical average of the risk $\widehat{R}_\mu(t;h)$ over 400 replications.



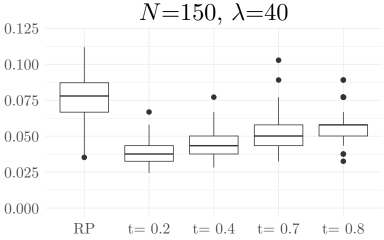
(N,λ) : —— $(1000,1000)$, −·−· $(1000,40)$, – – – $(150,40)$, ······ $(400,300)$.

Numerical illustration : mean estimation accuracy

MSE ratio of our mean estimator to RP, based on 400 replications.

N	λ	$t = 0.2$	$t = 0.4$	$t = 0.7$	$t = 0.8$
150	40	0.97	0.93	0.95	0.95
1000	40	0.97	0.94	0.92	0.92
400	300	1.01	0.97	1.00	0.99
1000	1000	0.99	1.00	0.99	1.00

Bandwidths for mean selected by RP and our local method.



In real data, our estimator captures μ's local irregularities better than RP.

Outline

Step 3 : Adaptive Prediction for Weakly Dependent FTS

The BLUP combines μ, Γ_ℓ, and the conditional variance of measurement errors.

> Goal : local adaptive estimation of the **BLUP** by **plugging** in the adaptive estimators of μ and Γ_ℓ.

Related works :

- Rubin & Panaretos (2020) : functional data recovery using a plug-in BLUP ; μ and Γ_ℓ estimated by pooling all observed points.
- Yao et al. (2005) : PACE method ; μ and Γ_0 estimated by pooling all observed points.

Best Linear Unbiased Predictor with lag $L = 1$

Let $t \in I$, $n_0 \geq 2$, and

$$\mathbb{Y}_{n_0} = (Y_{n,i}, n \in \{n_0 - 1, n_0\}, i = 1 \dots M_n)^\top, \qquad \mathbb{M}_{n_0} = \mathbb{E}_{M,T}(\mathbb{Y}_{n_0}).$$

Proposition 3.1. (Robinson, 1991)

The BLUP of $X_{n_0}(t)$ given $\mathbb{Y}_{n_0}$, is given by

$$\mathfrak{X}_{n_0}(t) = \mu(t) + B^\top(\mathbb{Y}_{n_0} - \mathbb{M}_{n_0}),$$

where $B = \text{Var}_{M,T}(\mathbb{Y}_{n_0})^{-1}\,\text{Cov}_{M,T}(\mathbb{Y}_{n_0}, X_{n_0}(t))$.

The predictor of $X_{n_0}(t)$ is $\widehat{\mathfrak{X}}_{n_0}(t)$, obtained by plugging in the adaptive estimators of μ, Γ_0, Γ_1, and σ :

$$\widehat{\mathfrak{X}}_{n_0}(t) = \widehat{\mu}(t) + \widehat{B}^\top(\mathbb{Y}_{n_0} - \widehat{\mathbb{M}}_{n_0}).$$

Confidence bands

Let the FTS $\{X_n\}$ and measurement errors $\{\varepsilon_{n,i}\}$ be Gaussian.

A $(1-\alpha)$ pointwise confidence interval for $X_{n_0}(t)$ is

$$\mathfrak{X}_{n_0}(t) \pm \Phi^{-1}(1-\alpha/2)\sqrt{\Sigma(t,t)},$$

where

- $\Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal law,
- $\Sigma(t,t)$ is the mean square error of $\mathfrak{X}_{n_0}(t)$.

Conjecture

The intervals form a smooth confidence band, with plug-in estimator

$$\widehat{\mathfrak{X}}_{n_0}(t) \pm \Phi^{-1}(1-\alpha/2)\sqrt{\widehat{\Sigma}(t,t)}.$$

✓ Uniform convergence of $\widehat{\mu}_N$ and $\widehat{\Gamma}_\ell$

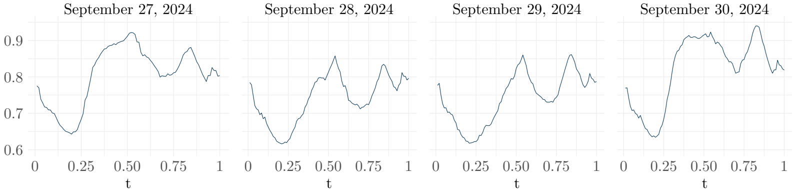
✗ Dimension of B tends to infinity, with $\lambda := \lambda_N$ as $N \to \infty$.

Numerical illustration : simulation setting

Settings : FAR(1) with parameters μ, ψ, H_t, L_t^2 and σ learned from real data

- Daily France electricity consumption, June–Sep 2024, 96 obs. per day.

Last 4 curves of **learning data**.



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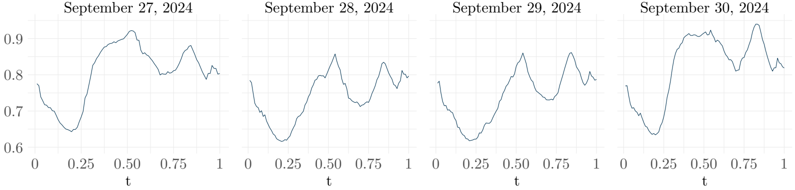
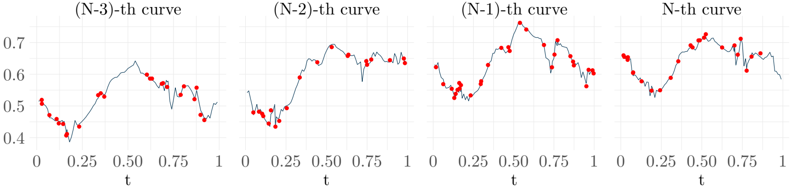


Last 4 curves of **simulated data**. Dots are noisy observations.



Numerical illustration : BLUP computation

The BLUP involves evaluating Γ_ℓ over many pairs (s, t).

- For each (s, t), $\widehat{R}_{\Gamma_\ell}$ is computed over a bandwidth grid to select $(h_s^\star, h_t^\star)$.

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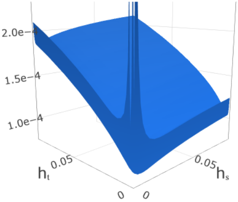
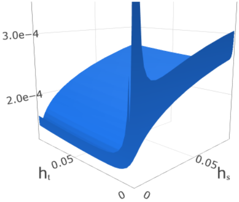
Empirical average of $\widehat{R}_{\Gamma_1}(s, t; h_s, h_t)$ over 400 replications.

$(s, t) = (0.1, 0.4)$
$(H_s, H_t) = (0.529, 0.527)$



$(h_s^\star, h_t^\star) = (0.003, 0.007)$

$(s, t) = (0.5, 0.2)$
$(H_s, H_t) = (0.784, 0.199)$



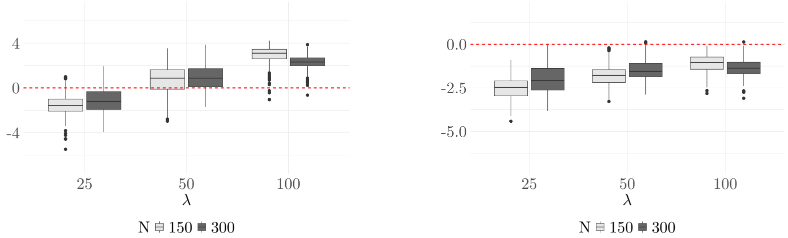
$(h_s^\star, h_t^\star) = (0.021, 0.001)$

Convex in (h_s, h_t) ; difference between $h_s^\star$ and $h_t^\star$ increases with $|H_s - H_t|$.

Numerical illustration : BLUP estimation accuracy

BLUP accuracy w.r.t. RP using log-ISE ratio over 400 replications :

$$\text{Ratio}^{(k)} = \frac{\int_0^1 \left(\widehat{\mathfrak{X}}_{n_0}^{(k)}(t) - X_{n_0}^{(k)}(t)\right)^2 dt}{\int_0^1 \left(\widehat{\mathfrak{X}}_{\text{RP}}^{(k)}(t) - X_{n_0}^{(k)}(t)\right)^2 dt}.$$

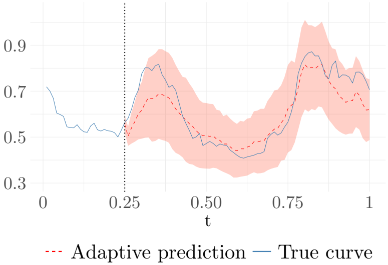


All points of the n_0^{th} curve observed — Only points in $(0, 1/2]$ of the n_0^{th} curve.

Numerical illustration : forecasting hydro-electricity production

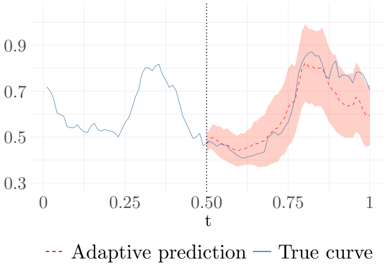
Data : Daily France hydro-electricity production, from Nov. 2023 to Mar. 2024, with a common design of 96 observations per day.

Only points in $(0, 1/4]$ of the n_0^{th} curve.



With 95% confidence bands

Only points in $(0, 1/2]$ of the n_0^{th} curve.



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Outline

Step 4 : Forecasting Wind and Photovoltaic Production

Goal : **fit** the **adaptive BLUP** and **compare** its performance with **alternative methods** for **one-step-ahead** curve prediction.

Related work :

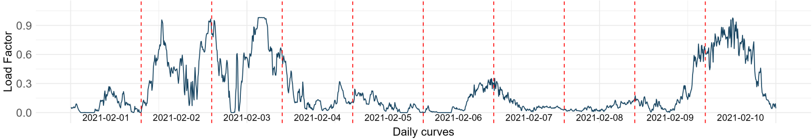
- Antoniadis *et al.* (2012) : French electricity demand forecasting ; handle non-stationarity by grouping days into stationary classes.
- Elías *et al.* (2022) : predict new curves using nearest neighbours.
- Aue *et al.* (2015) : FPCA-based prediction via truncated Karhunen–Loève expansion.

The data

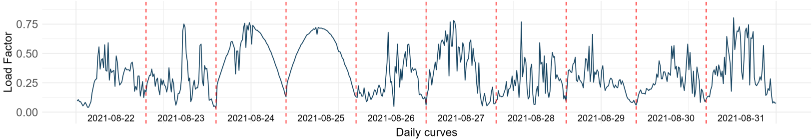
Selected **two Wind** and **two PV** farms, one each in the **north** and **south**.

- **W-North & W-South** : 151 curves, 144 obs. each (Nov 2020–Mar 2021).
- **PV-North & PV-South** : 153 curves, 61 obs. each (Apr–Aug 2021).

(a) First 10 daily curves of W-North in February 2021



(b) Last 10 daily curves of PV-North in August 2021, 9am–7pm each



Stationarity assumption

Stationary analyses (curve shapes, functional ACF, tests) show that the **FTSs are not stationary**, due to strong weather dependence.

Following Antoniadis *et al.* (2012), we assume **stationarity within groups of days** sharing similar weather patterns.

- Wind farms : cluster days using wind speed and gust speed.
- PV farms : cluster days using solar irradiance, cloud cover, and humidity.

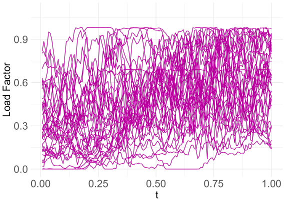
In practice, Météo-France forecasts provide the next day's weather :

- allowing us to identify its corresponding class,
- and use only curves from that class for forecasting.

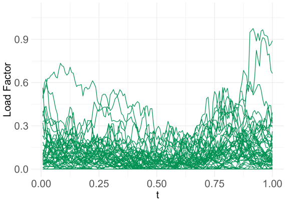
Example of day classes for a Wind farm

Curves varies around a mean, and the test does not reject stationarity.

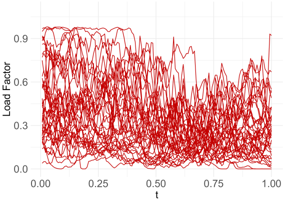
(a) Curves of class 1 of W-North



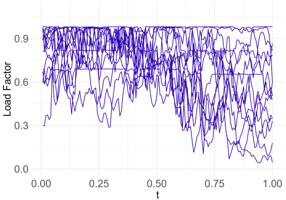
(b) Curves of class 2 of W-North



(c) Curves of class 3 of W-North



(d) Curves of class 4 of W-North



Performance comparison metric

Percentage GAIN in RMSE of adaptive BLUP w.r.t. Meth :

$$\text{GAIN} = 100 - \frac{\sqrt{\sum_{i=1}^{\lambda}\{Y_{N+1,i} - \widehat{\mathfrak{X}}_{N+1}(t_i)\}^2}}{\sqrt{\sum_{i=1}^{\lambda}\{Y_{N+1,i} - \widehat{\mathfrak{X}}_{\texttt{Meth}}(t_i)\}^2}} \times 100,$$

where $\{Y_{N+1,i}\}$ are the observations of the held-out curve, and Meth includes :

- Naïve : mean of last five observations, Mean-LF,
- Statistical : LM, ARMA,
- ML : RF, XGBoost, Prophet, LSTM, Deep-LSTM,
- FTS : FAR-Besse, FAR-Aue, EP, fKNN.

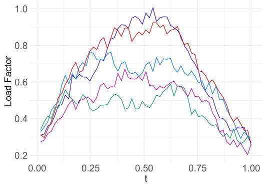
For Meth requiring covariates, **day** n**'s curve** and **time covariates** are used to **predict day** $n+1$.

Simulation study : impact of stationarity on BLUP accuracy

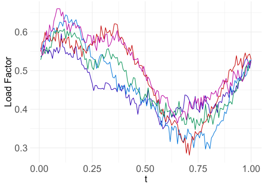
Settings : FAR(1) with parameters learned from daily Wind and PV curves.

Results : adaptive BLUP outperforms all alternatives.

(a) 5 last curves of PV-Simulated



(b) 5 last curves of W-Simulated



GAIN for a one-step-ahead prediction.

	W-Simulated	PV-Simulated
Mean-LF	50.89	54.50
LM	57.22	83.39
ARMA	49.08	61.56
RF	37.06	49.63
XGBoost	26.09	48.68
Prophet	45.26	40.39
LSTM	18.76	25.76
Deep-LSTM	31.59	08.18
FAR-Besse	52.33	68.83
FAR-Aue	11.59	02.54
EP	20.21	14.71
fKNN	33.23	50.56

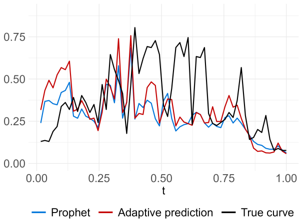
One-step-ahead PV production forecasting

Results : BLUP generally outperforms alternatives.

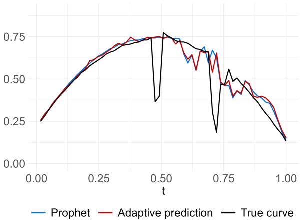
GAIN for a one-step-ahead prediction.

	PV-North	PV-South
Mean-LF	10.93	06.45
LM	35.69	29.75
ARMA	14.81	25.62
RF	02.42	06.94
XGBoost	08.21	20.82
Prophet	-02.09	03.80
LSTM	02.81	11.52
Deep-LSTM	03.97	13.05
FAR-Besse	03.51	04.21
FAR-Aue	-04.86	72.16
EP	-04.69	09.22
fKNN	05.48	26.20

BLUP vs Prophet pred. for PV-North



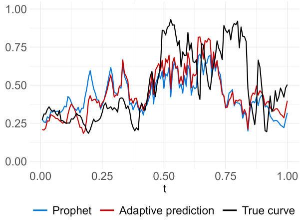
BLUP vs Prophet pred. for PV-South



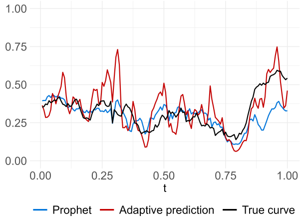
One-step-ahead Wind production forecasting

Results : BLUP generally outperforms alternatives.

BLUP vs Prophet pred. for W-North



BLUP vs Prophet pred. for W-South



GAIN for a one-step-ahead prediction.

	W-North	W-South
Mean-LF	30.33	41.87
LM	32.98	20.16
ARMA	15.42	36.50
RF	23.90	70.44
XGBoost	36.33	72.86
Prophet	06.75	-12.54
LSTM	32.25	63.57
Deep-LSTM	09.86	54.22
FAR-Besse	17.23	17.00
FAR-Aue	22.49	35.75
EP	36.89	06.68
fKNN	39.52	16.95

Conclusion

Study flexible FTS models :

1. Estimate local regularity
2. Adaptively estimate mean and (auto)covariances
3. Predict adaptively via a plug-in BLUP

Perspectives :

- Improve the study of BLUP consistency (Yao *et al.*, 2005)
- Incorporate covariates in the predictor to handle non-stationarity.

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Thanks for your attention !

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