DATA STORM

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Learning the Smoothness of Weakly Dependent Functional Times Series

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Motivation

We aim to estimate the local regularity parameters of the trajectories for FTS in the context of weak dependency.

Using dependent curves measured with noise at random discrete domain points, our goal is to build adaptive estimation of :

- mean and covariance functions,
- auto-covariance function,
- depth functions, *etc*.

The concept of local regularity, considered by Golovkine et al., (2022) for i.i.d. functional data, allows such constructions.

Weak Dependency

Let $\{X_n\}_{n\in\mathbb{Z}}$ be a stationary FTS, with continuous paths, on $I = [0,1]$:

- $(\mathcal{H}, \langle\cdot,\cdot\rangle_{\mathcal{H}})$: space of square integrable functions ;
- $(\mathcal{C}, \|\cdot\|_\infty)$: space of continuous functions on I.

The space $\mathbb{L}^p_{\mathcal{C}}$ is the space of $\mathcal{C}-$valued random element X such that

$$\nu_p(X) = \left(\mathbb{E}\left[\|X\|^p_\infty\right]\right)^{1/p} < \infty.$$

Weak dependency assumption : $\{X_n\}_n$ is $\mathbb{L}^{\mathbf{p}}_{\mathcal{C}}$ **– m-approximable**.

$\mathbb{L}^{\mathbf{p}}$ **– m**-approximation for functional data was introduced by Hörmann and Kokoszka (2010). Here we use $\|\cdot\|_\infty$ instead of $\|\cdot\|_{\mathcal{H}}$.

Local Regularity Parameters

The process X admits a *local regularity* at $t \in I$, with local exponent $H_t \in (0,1)$ and Hölder constant $L_t > 0$, if

$$\mathbb{E}\left[(X(u) - X(v))^2\right] \approx L_t^2 |u - v|^{2H_t},$$

for all $u, v \in [t - \Delta/2, t + \Delta/2]$ for some $\Delta > 0$.

We use some nonparametric estimates $\widetilde{X}_n$ to recover the X_n's.

For any u, v close to t, let

$$\widehat{\theta}(u,v) = \frac{1}{N} \sum_{n=1}^{N} \left\{\widetilde{X}_n(v) - \widetilde{X}_n(u)\right\}^2.$$

Our estimators of H_t and L_t^2 are defined as empirical counterparts of their respective definition.

Contribution and Perspectives

- Exponential bound for the concentration of the estimators of H_t and L_t^2 with FTS.
- Empirical investigation.
- Next steps : derive adaptive estimators for FTS analysis.

Thank you for Attention !

See you at the Poster Presentation for more details ;-)